

## Neutrino Flight Times in Cosmology

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If neutrinos have a small but non-zero mass, time-of-flight effects for neutrino bursts from distant sources can yield information on the large-scale geometry of the universe, the effects being proportional to  $\int a dt$ , where  $a$  is the cosmological expansion parameter. In principle absolute physical determinations of the Hubble constant and the acceleration parameter are possible. Practical realization of these possibilities would depend on neutrino masses being in a favorable range and the future development of very large detectors.

The existence of a small but non-zero mass for the neutrino, combined with very long flight times from astronomical sources, offers the possibility for some fundamental investigations, such as tests of the assumptions of relativity [1]. There is increasing evidence [2] for the existence of a small neutrino mass, and here we would like to note the further possibility of using such effects to study the parameters of cosmology. We require the detection of neutrino bursts from events at cosmological distances. This appears very difficult and may never be feasible. On the other hand we would have, along with such classic measures as the dependence on distance of the apparent brightness of “standard candles” or the angular size of “standard measuring rods”, a totally different method for investigating the large scale geometry of the universe. The method might be called “physical” as opposed to “geometric” and would not be subject to questions concerning evolutionary effects in the way that the “standard” candles and rods are.

As opposed to an exactly massless particle, which always travels on the light cone, a neutrino with a small mass has a velocity deviating from the speed of light in a way which depends on the cosmological epoch. Hence the delay in arrival time of the neutrino compared with a photon emitted in the same event, or the relative arrival times among neutrinos of different masses emitted in the same event, contains information on the cosmological epochs through which the neutrino has passed.

This is seen in the standard FRW metric [3]  $ds^2 = dt^2 - a^2(t)(d\mathbf{x})^2$ , where we define  $a(t)$  to be the expansion factor of the universe normalized to its present value:  $a(t) = R(t)/R(now)$ , so that  $a(now) = 1$ . We proceed by finding an equation for the coordinate velocity  $dx^i/dt$ , where  $x^i$  is along the particle’s flight direction. First we express  $dx^i/dt$  in terms of  $P^i(t)$ , the spatial part of the contravariant four-momentum  $m dx^\mu/ds$ . From the definition of the metric we have  $a(t)dx^i/dt = [a(t)P^i(t)]/\sqrt{m^2 + [a(t)P^i(t)]^2}$ .

Expanding for the relativistic case  $P \gg m$  we obtain  $a(t) dx^i/dt \approx 1 - \frac{1}{2}m^2/[a(t)P^i(t)]^2$ . To find  $P^i(t)$ , we now make use of the fact that the covariant or “canonical momentum”  $P_i$  is constant (since nothing depends on the  $\mathbf{x}$ -coordinate). Furthermore, since the different kinds of momenta are related through the metric tensor, they all become equal at  $t(now)$ , where  $a = 1$ . Hence we can identify the constant covariant momentum as  $P(now)$ , the momentum at the detector. Thus from  $P^i = g^{ij}P_j = 1/(a^2)P_i$ , we obtain  $P^i = 1/(a^2)P(now)$ . Thus we finally have

$$\frac{dx}{dt} \approx \frac{1}{a(t)} - a(t)\frac{1}{2}\frac{m^2}{P^2(now)}. \quad (1)$$

Or introducing  $\Delta x$  for the difference in the  $x$  coordinate of two different particles of mass  $m_2$  and  $m_1$  emitted in the same event

$$\frac{d(\Delta x)}{dt} \approx a(t)\frac{1}{2}\left[\frac{m_1^2}{P_1^2(now)} - \frac{m_2^2}{P_2^2(now)}\right] \quad (2)$$

At the present epoch with  $a = 1$ ,  $\Delta x$  is just the spatial separation of the two particles. Integrating, we have for this separation, or in view of  $v \approx c = 1$  for the time difference in arrival at a detector

$$\Delta t \approx \Delta x \approx \int a(t) dt \frac{1}{2}\left[\frac{m_1^2}{P_1^2(now)} - \frac{m_2^2}{P_2^2(now)}\right]. \quad (3)$$

A somewhat different effect follows if we consider a single mass state, or equivalently if the mass differences in question are undetectably small. Given a non-zero mass there is still a time-of-flight effect due to the variation of

velocity with energy. This was the original observation of Zatsepin [4] at the beginning of this subject, namely that in a burst there is a range of energies in general and therefore a non-zero mass for the neutrino leads to a dispersion of velocities, and so there is a spreading as the burst travels. For two momenta  $P(now)$  and  $P'(now)$  we have

$$\Delta t \approx \int a(t) dt \frac{1}{2} m^2 \left[ \left( \frac{1}{P(now)} \right)^2 - \left( \frac{1}{P'(now)} \right)^2 \right]. \quad (4)$$

Thus the spreading effect is governed by the same cosmological factor  $\int a dt$  as the time delay. The integral over  $t$  is from the time of emission to the present (“ $t(now)$ ”) and contains information on the cosmology. With the expansion [3] of  $a(t)$  for recent epochs  $a(t) = 1 + H[t - t(now)] - \frac{1}{2}qH^2[t - t(now)]^2 + \dots$ , and the redshift parameter  $z = 1/a - 1 = -H[t - t(now)] + (1 + q/2)H^2[t - t(now)]^2 + \dots$ , we can give the results of the integration in terms of the directly observable  $z$ , in a low  $z$  approximation to order  $z^2$ , in terms of the present Hubble constant  $H$  and acceleration parameter  $q$ :

$$\Delta t \approx \frac{z}{H} \left[ 1 - \frac{3+q}{2}z + \dots \right] \frac{1}{2} \left[ \frac{m_1^2}{P_1^2(now)} - \frac{m_2^2}{P_2^2(now)} \right] \quad (5)$$

while for the burst spreading we have

$$\Delta t \approx \frac{z}{H} \left[ 1 - \frac{3+q}{2}z + \dots \right] \frac{1}{2} m^2 \left[ \left( \frac{1}{P(now)} \right)^2 - \left( \frac{1}{P'(now)} \right)^2 \right]. \quad (6)$$

The first thing to notice about these formulas is that they provide a direct physical determination of  $H$ , the Hubble constant. Knowing the neutrino mass or masses, the neutrino energy and the redshift of the source, measurement of a  $\Delta t$  determines  $H$ . It would certainly be interesting to compare a determination of  $H$  in this “absolute” way with the classical astronomical results.

Observe that it is not necessary to know the distance to the source,— or rather a “distance” is determined through the particle velocities and their  $\Delta x$  at the detector. Thus in addition to other classical distance measures in cosmology, like the “luminosity distance” [3], we may say that there is another, the “neutrino” or “flight time” distance,  $d_\nu$ , defined through  $\Delta x = \Delta v d_\nu$ , where  $\Delta v$  is  $\frac{1}{2} \left[ \frac{m_1^2}{P_1^2(now)} - \frac{m_2^2}{P_2^2(now)} \right]$ , or  $\frac{1}{2} m^2 \left[ \left( \frac{1}{P(now)} \right)^2 - \left( \frac{1}{P'(now)} \right)^2 \right]$ .

The cosmic acceleration  $q$  appears in the quadratic correction. Here again, knowing the neutrino masses, their energy, redshift  $z$  of the source, and the Hubble constant, we could—in principle—find the acceleration parameter from just one event. Recent reports [5] of a non-zero value of  $q$  from a “standard candle” method using distant supernovas has attracted considerable attention. It would be of the greatest interest if a totally different method could confirm these results.

Although we have carried out the above arguments for the spatially flat ( $k = 0$ ) FRW metric, it may be verified that due to the existence of a similar constant “canonical momentum” in the other, spatially curved ( $k = \pm 1$ ) FRW metrics, the same arguments go through with the same results.

The most straightforward application of these ideas would be to situations where bursts of photons ( $m = 0$ ) or different neutrino mass states, arriving distinctly separated in time, are detected. Assuming the neutrino masses have been well established by the time such detections become feasible, two well-measured events whose redshifts can be determined and Eq.(5) would determine the Hubble constant and the acceleration parameter. Unfortunately, the observation of events with the different mass components well separated seems unlikely. Given that we expect neutrino masses in the eV range or less, we can express our basic flight time factor as

$$\frac{(m/\text{eV})^2}{2(P/\text{GeV})^2} \approx 50 \mu\text{sec}/\text{Mpc} \quad (7)$$

It appears that even at a thousand Mpc, a substantial part of the way across the visible universe, we may only expect some msec delays. There is of course the chance, not yet completely excluded, that the heavier neutrino masses are in the range of some tens of eV, which could lead to delays of minutes, for the most distance sources at GeV energies. Although  $\mu\text{sec}$  timing or better is well within the capabilities of photon and neutrino detectors, the duration of the production event and its fluctuations are likely to involve much longer time scales. For example gamma-ray bursts or supernova neutrino bursts last on the order of several seconds. So, unless heavier neutrinos and/or lower energies are accessible, this will make the direct observation of the relative delay of different neutrino mass states or the delay relative to a photon signal difficult to extract [6]. A detailed analysis involving high statistics, with sources at various distances and theoretical modelling of the production event would probably be necessary.

In view of these probable difficulties, it is important to note that the pulse spreading and related effects represented by Eq.(6) offer potentially observable time-of-flight effects involving a cosmological factor, even if distinct masses

cannot be resolved. However, unless the event is intrinsically very short, the modelling of the event and the assumption that a certain type of event is the same at different epochs become necessary. We should stress that Eq.(6) only represents travel time effects on the width of the burst, these will in general have to be folded with the red shift of the intrinsic time scale of the event itself. Note, however, that the latter  $\propto z$  only and do not involve a quadratic term.

One may even carry this kind of speculation a step further and consider the cosmology of neutrino oscillations. That is, we can consider neutrino mass differences so extremely small that coherent oscillation phenomena are possible over cosmological distances [7]. Although not very likely of practical realization, the question involves some interesting conceptual issues and will be presented separately [8].

It is of course intriguing and of conceptual interest that there is a different, “non-geometric” way of investigating cosmology. As a practical matter, because of the small rate of neutrino interactions and the reduction of the flux with distance, the only hope that these speculations might become reality would seem to lie with the further development of the extremely big, Km size detectors. Since time-of-flight effects decrease with increasing particle energy, for the time delay and burst spreading effects, lower neutrino energies, and instrumentation of detection systems for lower neutrino energies, are favored. Unfortunately detection cross sections are smaller at lower energy, so even larger detectors would seem to be called for. On the other hand, for the hypothetical oscillation effects alluded to above, higher energies seem to be indicated [8]; there cross section are larger and may tend to compensate for the general decrease of flux with energy.

A fascinating aspect of studying cosmology by these neutrino-based techniques, should it ever be possible, is that the “look back time” is potentially much deeper than with optical methods. A neutrino burst—highly redshifted—can reach us essentially undisturbed after neutrino decoupling, some minutes into the Big Bang. If such bursts exist (say from highly overdense regions collapsing to black holes) the present methods could allow us to determine the cosmology back to these early epochs. At such large values of  $z$ , of course, the above small  $z$  approximations would have to be replaced by expressions involving the full cosmological model, as would also be necessary for other high  $z$  observations. If even more weakly interacting and non-zero mass but highly relativistic particles than the usual neutrinos exist (say right-handed neutrinos) the “look back time” goes deeper but the detection difficulties get worse.

The only presently known candidates at present for our procedures are the photon and the neutrinos. Naturally, if heavier but neutral and sufficiently long-lived objects should exist and be emitted in cosmic events, (say the WIMP of dark matter searches) there would be a substantial enhancement for time-of-flight effects. I am thankful to V. Zakharov for this last remark and G. Raffelt for helpful comments on the manuscript.

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